EXERCISE – V

HINTS & SOLUTIONS

Sol.1 **(a)** Given $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1)\end{vmatrix}$

Applying $C_3 \to C_3 - (C_1 + C_2)$

$$= \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0\end{vmatrix} = 0$$

$\therefore \quad f(x) = 0 \Rightarrow f(100) = 0$

(b) Given system of equation,
$u + 2v + 3w = 6$
$4u + 5v + 6w = 12$
$6u + 9v = 4$

Augmented matrix, $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 4 & 5 & 6 & 12 \\ 6 & 9 & 0 & 4\end{array}\right)$

Applying $R_2 \to R_2 - 4R_1$ & $R_3 \to R_3 - 6R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -3 & -6 & -12 \\ 0 & -3 & -18 & -32\end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -3 & -6 & -12 \\ 0 & 0 & -12 & -20\end{array}\right)$$

$\Rightarrow u + 2v + 3w = 6$
$-3v - 6w = -12$
$-12\,w = -20$

$\therefore\ u = \left(\dfrac{-1}{3}\right); v = \dfrac{2}{3}; w = \dfrac{5}{3}$

$\Rightarrow (u + v + w) = 2\ ;\ \left(\dfrac{1}{u}+\dfrac{1}{v}+\dfrac{1}{w}\right) = \dfrac{9}{10}$

Now, a, b, c, d are in GP, then
$b^2 = ac;\ c^2 = bd;\ ad = bc$
$\therefore\ [(b-c)^2 + (c-a)^2 + (d-b)^2]$
$= b^2 + c^2 + c^2 + a^2 + d^2 + b^2 - 2bc - 2ca - 2bd$
$= (a-d)^2$
Observing given equation,

$$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2+[(b-c)^2+(c-a)^2+(d-b)^2]\,x+(u+v+w)=0$$

& $20\,x^2 + 10\,(a-d)^2\,x - 9 = 0$
we can say, equations have reciprocal roots.

Sol.2 Since, the given system has non-zero solution

$$\therefore \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1\end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1+k & -k-1 & -1 \\ 1+k & -2 & -1 \\ 0 & 0 & -1\end{vmatrix} = 0$$

($\because$ Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 + C_3$)
$\Rightarrow\ 2(k+1) - (k+1)^2 = 0 \Rightarrow (k+1)\,(2-k-1) = 0$
$\Rightarrow\ k = \pm 1$

Sol.3 Let $\Delta = \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$

Applying $R_2 \to R_2 + R_3$

$$=\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right)+\sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)+\sin\left(2+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

$$=\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ 2\sin\theta\cos\frac{2\pi}{3} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin 2\theta\cos\frac{4\pi}{3} \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

$$=\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix} = 0$$

(since R_1 and R_2 are proportional)

Sol.4 For non – trivial solution,

$$D = \begin{vmatrix} 2r & -2 & 3 \\ 1 & r & 2 \\ 2 & 0 & r\end{vmatrix} = 0$$

$\Rightarrow 2r\,(r^2 - 0) + 2(r - 4) + 3(0 - 2r) = 0$
$\Rightarrow r = 2$
$\therefore$ system of equations become,
$4x - 2y + 3z = 0$ ....(1)
$x + 2y + 2z = 0$ .....(2)
$2x + 2z = 0$ ....(3)
Let $x = k$
from equation (3),
$z = (-k)$ .....(4)
from equation (1) & (4),

$y = \dfrac{k}{2}$

Sol.5 Given : $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$

$\Rightarrow a^2$ [sin nx. cos (n – 1)x – cos nx. sin (n – 1)x]
– a [sin (n + 1)x. cos (n – 1)x – cos (n + 1)x. sin (n – 1)x]
+1 [cos nx. sin (n + 1)x – sin nx. cos (n + 1) x] = 0
$\Rightarrow a^2$ sin [nx – (n – 1)x] – a sin [(n + 1)x – (n – 1) x]
+ sin [(n + 1) x – nx] = 0
$\Rightarrow$ sin x (a^2 – a cos x + 1) = 0 $\Rightarrow$ sin x = 0
$\Rightarrow x = n\pi;\ n \in I$

Sol.6 Augmented matrix,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & \lambda \\ 3 & \lambda+2 & -3 & 2\lambda+1 \end{array}\right) \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & \lambda-1 \\ 0 & \lambda-1 & -6 & 2\lambda-2 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & -3 & \lambda-1 \\ 0 & \lambda-5 & 0 & 0 \end{array}\right)$$

Hence, consistent for all values of λ,

Case – I : When $\lambda = 5$,
x + y + z = 1
2y – 3z = 4
Let z = k

$\Rightarrow$ 2y – 3k = 4 $\Rightarrow$ y = $\left(\dfrac{3k+4}{2}\right)$

& x = 1 – $\dfrac{3k+4}{2}$ – k $\Rightarrow$ x = $\left(\dfrac{-5k-2}{2}\right)$

Case – 2 : when $\lambda \neq 5$
x + y + z = 1
2y – 3z = λ – 1
(λ – 5)y = 0 $\Rightarrow$ y = 0

$\Rightarrow$ –3z = λ – 1 $\Rightarrow$ z = $\left(\dfrac{1-\lambda}{3}\right)$

& x = 1 – $\dfrac{1-\lambda}{3}$ $\Rightarrow$ x = $\left(\dfrac{2+\lambda}{3}\right)$

Sol.7 Given, $\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} a^2x-aby-ac & bx+ay & cx+a \\ abx+a^2y & -ax+by-c & cy+b \\ acx+a^2 & cy+b & -ax-by+c \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + bC_2 + cC_3$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} (a^2+b^2+c^2) & ay+by & cx+a \\ (a^2+b^2+c^2)y & by-c-ax & b+cy \\ (a^2+b^2+c^2) & b+cy & c-ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} x & ay+bx & cx+a \\ y & by-c-ax & b+cy \\ 1 & b+cy & c-ax-by \end{vmatrix} = 0$$

($\because$ $a^2 + b^2 + c^2 = 1$ given)

Applying $C_2 \to C_2 + bC_1$ and $C_3 \to C_3 - cC_1$

$$\Rightarrow \frac{1}{a}\begin{vmatrix} x & ay & a \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{ax}\begin{vmatrix} x^2 & axy & ax \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + yR_2 + R_3$

$$\Rightarrow \frac{1}{ax}\begin{vmatrix} x^2+y^2+1 & 0 & 0 \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix} = 0$$

$\Rightarrow \dfrac{1}{ax}$ [($x^2 + y^2$ + 1) {(–c –ax)(–ax – by) –b(y)}] = 0

$\Rightarrow \dfrac{1}{ax}$[($x^2 + y^2$ + 1)(acx + bcy + a^2x^2 + abxy – bcy)] = 0

$\Rightarrow \dfrac{1}{ax}$ [($x^2 + y^2$ + 1)(acx + a^2x^2 + abxy)] = 0

$\Rightarrow \dfrac{1}{ax}$ [ax ($x^2 + y^2$ + 1)(c + ax + by)] = 0

$\Rightarrow$ ($x^2 + y^2$ + 1) (ax + by + c) = 0
$\Rightarrow$ ax + by + c = 0
which represent a straight line.

Sol.8 For infinitely many solutions, we must have,

$$\frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$$

$\Rightarrow$ k = 1

Sol.9 Given $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, abc = 1 and $A^TA = I$ ....(1)

Now $A^TA = I$

$$\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow a^2 + b^2 + c^2 = 1$ and $ab + bc + ca = 0$ ....(2)

we know $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow a^3 + b^3 + c^3 = (a + b + c)(1 - 0) + 3$

(from equation (1) and (2))

$\Rightarrow a^3 + b^3 + c^3 = (a + b + c) + 3$ .....(3)

Now $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$ ....(4)

$\therefore$ From equation (3), $a^3 + b^3 + c^3 = 1 + 3 = 4$

Sol.10 Given $2x - y, + 2z = 2$

$x - 2y + z = -4$

$x + y + \lambda z = 4$

since, given system has no solution

$\therefore \Delta = 0$ and any one amongst $\Delta_x, \Delta_y, \Delta_z$ is non-zero

Let $\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$ and $\Delta_z = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & 4 \end{vmatrix}$

$= 6 \neq 0$ $\Rightarrow \lambda = 1$

Sol.11 Given $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$

$$= \begin{bmatrix} \alpha^2+4 & 4\alpha \\ 4\alpha & \alpha^2+4 \end{bmatrix}\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \alpha^3+12\alpha & 6\alpha^2+8 \\ 6\alpha^2+8 & \alpha^3+12\alpha \end{bmatrix}$$

then $|A^3| = 125$

$$\Rightarrow \begin{vmatrix} \alpha^3+12\alpha & 6\alpha^2+8 \\ 6\alpha^2+8 & \alpha^3+12\alpha \end{vmatrix} = 125$$

$\Rightarrow (\alpha^3 + 12\alpha)^2 - (6\alpha^2 + 8)^2 = 125$

$\Rightarrow (\alpha^3 + 6\alpha^2 + 12\alpha + 8)(\alpha^3 - 6d^2 + 12\alpha - 8) = 125$

$\Rightarrow (\alpha + 2)^3 (\alpha - 2)^3 = 125$

$\Rightarrow [(\alpha + 2)(\alpha - 2)]^3 = (5)^3 \Rightarrow \alpha^2 - 4 = 5$

$\Rightarrow \alpha^2 = 9$ $\Rightarrow \alpha = \pm 3$

Sol.12 Given: $|M| = 1$ & $M^T M = I$

Now, $|M^T| = |M| = 1$

$\therefore (M - I)^T (M^T - I^T) = (M^T - I)$

But, $M^T M = I$

$\therefore (M - I)^T = M^T - M^T M = M^T (I - M)$

Now,

$\det (M - I)^T = \det (M^T (I - M))$

$\Rightarrow \det (M - I)^T = \det (M^T) \det (I - M)$

$\Rightarrow \det (M - I) = \det (M) \det (I - M)$

$\Rightarrow \det (M - I) = (-1) \det (M - I)$

$\Rightarrow 2 \det (M - I) = 0$

$\Rightarrow \det (M - I) = 0$

Sol.13 Since AX = U has infinitely many solutions

$\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{vmatrix} = 0$$

$\Rightarrow a(bc - bd) + 1(d - c) = 0$

$\Rightarrow (d - c)(ab - 1) = 0$

$\Rightarrow ab = 1$ or $d = c$

Again $|A_3| = \begin{vmatrix} a & 0 & f \\ 1 & c & g \\ 1 & d & h \end{vmatrix} = 0 \Rightarrow g = h$

$|A_2| = \begin{vmatrix} a & f & 1 \\ 1 & g & b \\ 1 & h & b \end{vmatrix} = 0 \Rightarrow g = h$

and $|A_1| = \begin{vmatrix} a & f & 1 \\ 1 & g & b \\ 1 & h & b \end{vmatrix} = 0 \Rightarrow g = h$

$\therefore g = h, c = d$ and $ab = 1$ .....(1)

Now BX = v $|B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix} = 0$

(since, c_2 and c_3 are equal) (from equation (1)

$\therefore$ Bx = V has no solution

$$|B_1| = \begin{vmatrix} a^2 & 1 & 1 \\ 0 & d & c \\ 0 & g & h \end{vmatrix} = 0$$

(since, c = d and g = h) (from equation (1)

$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2 cf = a^2 df$$

(since c = d)

since $adf \neq 0 \Rightarrow |B_2| \neq 0$

$\therefore |B| = 0$ and $|B_2| \neq 0$

$\therefore$ Bx = V has no solution

Sol.14 Every square matrix satisfied its charact eristic equation

i.e. $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & -2 & 4-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)\{(1-\lambda)(4-\lambda)+2\} = 0$

$\Rightarrow \quad \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Rightarrow \quad A^3 - 6A^2 + 11A - 6I = 0 \quad ....(1)$

Given, $6A^{-1} = A^2 + cA + dI$, multiplying both sides by A, we get

$$6I = A^3 + cA^2 + dA$$

$\Rightarrow \quad A^3 + cA^2 + dA - 6I = 0 \quad ....(2)$

On comparing equation (1) and (2) we get

$$c = -6 \text{ and } d = 11$$

Sol.15 Now $P^T P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$

$\Rightarrow P^T P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow P^T P = I \Rightarrow P^T = P^{-1}$

since $Q = PAP^T$

$\therefore P^T Q^{2005} P = P^T [(PAP^T)(PAP)^T)....2005 \text{ times}]P \quad ...(1)$

$$= \underbrace{(P^TP)A(P^TP)A(P^TP)....(P^TP)A(P^TP)}_{2005 \text{ times}}$$

$= IA^{2005} = A^{2005}$ (from equation (1))

$\therefore \quad A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

....

....

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix} \quad \therefore \quad P^T Q^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

Sol.16 **(a)** Let $U_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, so that $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x = 1, y = -2, z = 1$

$\therefore \quad U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

similarly $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$, $U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

Hence $U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$

$\therefore \quad |U| = 3$

(b) More over

$$\text{adj } U = \begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$$

$\therefore \quad U^{-1} = \dfrac{\text{adj } U}{3}$ and sum of the $U^{-1} = 0$

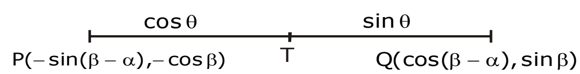
(c) The value of $[3\ 2\ 0]\, U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$= [3\ 2\ 0] \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$= [-1\ 4\ 4] \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$= [5]$

Sol.17 **(a)** Join P & Q,



Let T divides PQ in ratio $\cos\theta : \sin\theta$, then

$$\left(\frac{\cos(\beta-\alpha).\cos\theta - \sin(\beta-\alpha).\sin\theta}{\cos\theta + \sin\theta}, \frac{\cos\theta\sin\beta - \cos\theta - \sin\beta\sin\theta}{\cos\theta + \sin\theta}\right)$$

$\because$ P, T, Q are collinear

$\therefore$ P, Q, R are non – collinear.

(b) The given system of equation can be expressed as

$$\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$$

Applying $R_2 \to R_2 \to R_1$, $R_3 \to R_3 + R_1$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{vmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ z \end{bmatrix} \quad R_3 \to R_3 - R_2$$

when $k \neq 3$ the given system of equations has no solution

$\Rightarrow$ statement I is true clearly statement II is also true as it is rearrangement of rows and columns

of $\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -1 & 1 & -2 \end{bmatrix}$

Hence option (A) is correct

Sol.18 **(A)** Let $y = \dfrac{x^2+2x+4}{x+2}$

$\Rightarrow x^2 + (2 - y)x + 4 - 2y = 0$

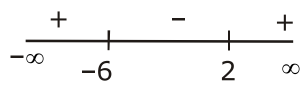
for real x, $D \geq 0$

$\Rightarrow (2 - y)^2 - 4(4 - 2y) \geq 0$

$\Rightarrow y^2 + 4y - 12 \geq 0$

$\Rightarrow (y + 6)(y - 2) \geq 0$

$\Rightarrow y \leq -6$ or $y \geq 2$



$\therefore$ Minimum Value = 2

(B) Given : $A^t = A$; $B^t = (-B)$

and $(A + B)(A - B) = (A - B)(A + B)$

$\Rightarrow A^2 + B^2 - AB + BA = A^2 + B^2 + AB - BA$

$\Rightarrow AB = BA$

Now, $(AB)^t = B^t A^t = -BA = -AB$

$\therefore$ k = 1, 3

(C) Given, $a = \log_3 \log_3 2$

$\Rightarrow \log_3 2 = 3^a$

$\Rightarrow 3^{-a} = \log_2 3$

Now, $1 < 2^{-k+3^{-a}} < 2$

$\Rightarrow 1 < 2^{-k} . 2\log_2 3 < 2$

$\Rightarrow 1 < 3. 2^{-k} < 2$

$\Rightarrow \dfrac{1}{3} < 2^{-k} < \dfrac{2}{3}$

$\Rightarrow \dfrac{3}{2} < 2^{k} < 3$

$\Rightarrow \log_2\left(\dfrac{3}{2}\right) < k < \log_2 3$

Solve & get value of k

(D) Given $\sin\theta = \cos\phi$

$\Rightarrow \cos\left(\dfrac{\pi}{2} - \theta\right) = \cos\phi \Rightarrow \left(\dfrac{\pi}{2} - \theta\right) = 2n\pi \pm \phi$

$\Rightarrow \left(\theta \pm \phi - \dfrac{\pi}{2}\right) = -2n\pi$

$\Rightarrow \dfrac{1}{\pi}\left(\theta \pm \phi - \dfrac{\pi}{2}\right) = -2n$

for n = 0; we get (0)

for n = 1; we get (–2)

for n = (–1); we get (2)

Sol.19 **(a)** Since A is a symmetric matrix and its five entries are 1 and 4 entries are zero. So, following cases are possible : -

(i) When 2 entries of principal diaagonal are zero:-

Total matrices = ${}^3C_2 \times {}^3C_1 = 3 \times 3 = 9$

(ii) If all entries of principal diagonal are 1

Total matrices = ${}^3C_2 = 3$

Hence, total matrices = 9 + 3 = 12

(b) For unique solution, $|A| \neq 0$; Let $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$

Possible matrices such that $|A| \neq 0$ are:

Case–1: $\begin{bmatrix} 0 & b & c \\ b & 0 & e \\ c & e & 1 \end{bmatrix} \Rightarrow$ for c = 0 or e = 0 $\Rightarrow |A| \neq 0$

Hence, 2 matrices are possible.

Case–2: $\begin{bmatrix} 1 & b & c \\ b & 0 & e \\ c & e & 0 \end{bmatrix} \Rightarrow$ for b = 0 or c = 0 $\Rightarrow |A| \neq 0$

Hence, 2 matrices are possible.

Case–3: $\begin{bmatrix} 0 & b & c \\ b & 1 & e \\ c & e & 0 \end{bmatrix} \Rightarrow$ for b = 0 or e = 0 $\Rightarrow |A| \neq 0$

Hence, 2 matrices are possible.

Case–4: $\begin{bmatrix} 1 & b & c \\ b & 1 & e \\ c & e & 1 \end{bmatrix}$

$\Rightarrow$ for b = c = 0, |A| = 0

for c = e = 0, |A| = 0

for b = e = 0, |A| = 0

Hence, no matrix is possible

$\therefore$ Total matrice = 2 + 2 + 2 = 6

(c) Six matrices (Augmented) for which |A| = 0

(i) $\left(\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \Rightarrow$ Inconsistent

(ii) $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \Rightarrow$ Inconsistent

(iii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right) \Rightarrow$ Infinite

(iv) $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow$ Inconsistent

(v) $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \Rightarrow$ Inconsistent

(vi) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \Rightarrow$ Infinite

Sol.20 Given system will give equation of 3 planes but 3 planes cannot intersect at two distinct points. Hence, no matrix possible.

Sol.22 After applying suitable transformation,

$|A| = (2k + 1)^3$ and $\underbrace{|B| = 0}_{\text{Skew symetric of odd order}}$

Now, $|\text{adj } A| = |A|^{n-1} = |A|^{3-1} = |A|^2$
Thus, det (adj A) + det (adj B) = 10^6
$\Rightarrow |A|^2 + |B|^2 = 10^6$ $\Rightarrow ((2k + 1)^3)^2 = 10^6$

$\Rightarrow 2k + 1 = 10$ $\Rightarrow k = \dfrac{9}{2} = 4.5$ $\Rightarrow [k] = 4$

Sol.23 Given: $M^T = (-M)$; $N^T = (-N)$; $MN = NM$
Now, $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$
$\Rightarrow M\, M\, N\, N\, (N^{-1})\, (M^T)^{-1}\, (N^{-1})^T\, M^T$
$\Rightarrow M\, N\, M\, I\, (M^T)^{-1}\, (N^T)^{-1}\, M^T$
$\Rightarrow -MN\, M^T\, (M^T)^{-1}\, (N^T)^{-1}\, M^T$ $\Rightarrow -M(N\ I)\, (N^T)^{-1}\, M^T$
$\Rightarrow M\, N^T\, (N^T)^{-1}\, M^T$ $\Rightarrow (M\ I)\, M^T$
$\Rightarrow -M\, M$ $\Rightarrow -M^2$

Sol.24 Let $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

Now, $M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow a_2 = (-1);\ b_2 = 2;\ c_2 = 3$

Now, $M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_1 & -1 & a_3 \\ b_1 & 2 & b_3 \\ c_1 & 3 & c_3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$\Rightarrow a_1 = 0$; $b_1 = 3$; $c_1 = 2$

Now, $M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & -1 & a_3 \\ 3 & 2 & b_3 \\ 2 & 3 & c_3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$\Rightarrow c_3 = 7$ $\therefore$ sum of diagonal elements
$= a_1 + b_2 + c_3 = 0 + 2 + 7 = 9$

Sol.25 Let $P = \begin{bmatrix} a & b & c \\ d & e & f \\ p & q & r \end{bmatrix}$

Now, $P = [a_{ij}]$ & $\theta = [b_{ij}]$ and $b_{ij} = 2^{i+j} a_{ij}$

$\therefore Q = \begin{bmatrix} 2^2 a & 2^3 b & 2^4 c \\ 2^3 d & 2^4 e & 2^5 f \\ 2^4 p & 2^5 q & 2^6 r \end{bmatrix}$

$\Rightarrow |Q| = 2^{12} |P| = 2^{12}.2 = 2^{13}$

sol.26 Let $P = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$P^T = 2P + I \Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ 2c_1 & 2c_2 & 2c_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} 2a_1 + 1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 + 1 & 2b_3 \\ 2c_1 & 2c_2 & 2c_3 + 1 \end{bmatrix}$

On comparing corresponding elements,
$a_1 = (-1)$; $b_2 = (-1)$; $c_3 = (-1)$
Also, $b_1 = c_1 = a_2 = c_2 = a_3 = b_3 = 0$

Hence, $P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = (-I)$

$\Rightarrow PX = (-I)X = (-X)$

Sol.27 As we know that,
$|\text{adj } A| = |A|^{n-1}$
Here, $|\text{adj } P| = |P|^{3-1} = |P|^2$

Now, $|\text{adj } P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$

$\Rightarrow |\text{adj } P| = 1(3 - 7) - 4(6 - 7) + 4\,(2 - 1)$
$\Rightarrow |\text{adj } P| = 4$ $\Rightarrow |P|^2 = 4$
$\Rightarrow |P| = \pm 2$ $\Rightarrow |P| = 2$ or $|P| = (-2)$